MATH 208 C — MIDTERM 1 — Autumn 2022

NAME (Please print): ______________________________

There are 4 problems. Show all of your work and justify your answers.

Problem	Score
1	
2	
3	
4	
Total	

- **All phones and headphones must be put away in your bag.**
- **No talking or looking around during the exam.** Any form of cheating will result in a zero on this exam.
- You are allowed one 8.5x11 sheet of notes (written on both sides) and a Texas Instruments TI-30X IIS calculator.

(1) Let S denote the set of solutions to the following system of linear equations:

$$\begin{array}{rcrcrcrcl} 2x_1 & + & x_2 & - & x_3 & + & x_4 & = & 0 \\ & - & x_2 & + & 2x_3 & + & x_4 & = & 0 \end{array}$$

(a) S is the intersection of planes.

(i) How many planes?

(ii) Where do the planes live?

(b) What do you expect S to look like? Why?

(c) Compute S.

(d) Using (c) find a set of vectors that span S. Explain what you did.

(e) How would you construct a new equation which can be added to the system so that the set of solutions does not change? Describe your plan in words and then write down such an equation.

(f) Using (c) can you write $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ as a linear combination of $\begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ in which every vector participates? Explain your strategy.

(2) (a) What is the span of the following vectors? Use Gaussian elimination to answer this question. Show all your work and box your answer.

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -7 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -15 \end{pmatrix}$$

(b) Looking at the echelon form you computed in (a) find as many linearly independent vectors as you can among the original four vectors. Explain how you used the echelon form to arrive at your conclusion.

(3) In each case below find the values of t (when possible) for which the given vectors are linearly **dependent**. Give reasons in each case.

(a) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \end{pmatrix}, \begin{pmatrix} t \\ 7 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ t \\ 5 \end{pmatrix}$

(c) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1-t \\ 1-t \\ -1+t \end{pmatrix}$

(d) $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} t \\ t \\ 2 \\ 2 \end{pmatrix}$

(4) Jake wants to find a polynomial function that passes through the points $(1,0),(2,0),(3,0)$ by solving a system of linear equations. In each case below,

(i) set up the equations he would solve,

(ii) decide whether he will be able to find such a polynomial, and

(iii) if so, how many.

No need to calculate the polynomial(s). In each question, mark the parts (i),(ii),(iii) clearly and give reasons for (ii) and (iii).

(a) A line $y = a_0 + a_1 x$.

(b) A quadratic $y = a_0 + a_1 x + a_2 x^2$.

(c) A cubic $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$